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Petri net dynamic interpretation

Yusupov R.[1], Sokolov B.[1], Zakharov V.[1*] and Semenov A.[1]

[1] St. Petersburg Federal Research Center of the Russian Academy of Sciences, 39, 14th Line V.O., St. Petersburg, 199178, Russia

[*]Corresponding author. Email address: valeriov@yandex.ru

Abstract

The main problem of modern modeling tools for the functioning of complex objects is the presence of an insurmountable semantic gap between graphical (analytical) and executable models of control and data processing processes. The results of previous studies have shown that with automatic targeted dynamic parallelization of processes, they achieve high efficiency of using the resulting models in the management of complex objects. The paper presents a number of new variants of models that allow us to constructively describe the natural parallelism of information processes (InP) and the corresponding control processes in the automated control system. The developed algorithms for finding optimal schedules and the corresponding plans for the functioning of funds can be used to find optimal rules for triggering transitions in Petri nets, evaluating the reachability of a given Petri net marking.

Keywords: parallelism of management and information processes; real concurrency; apparent concurrency

1. Introduction

In modern automated controlling systems for complex objects (ACS CO) the processes of obtaining, storing, processing and transmitting data, information and knowledge are closely intertwined with the processes of monitoring and managing these objects. At the same time, these processes can proceed both sequentially and in parallel, using various ACS CO resources. An object is called complex from an epistemological point of view if its cognition requires the joint involvement of many models, many theories, and in some cases, many scientific disciplines (interdisciplinary research), and the implementation in model representations of an installation for deep consideration of probabilistic and improbable uncertainties. The analysis shows that within the framework of existing approaches, when describing the processes under consideration, the dynamic nature of the partial order arising between them is not taken into account. Such a dynamically changing partial order in the works (Yatsutko and Vykhovanets 2013) is proposed to be called the "natural parallelism" of the processes of functioning of the ACS CO. In these works, it is shown that with automatic purposeful dynamic parallelization of processes, high efficiency is achieved in using the resulting models in the control of the CO. The implemented parallelism is close to the natural parallelism of the simulated physical and information processes. The execution of processes using dynamic parallelization is characterized by the possibility of scaling to a different number of parallel interpreters (processors, parallel threads). This paper presents a new approach to the formal description of the natural parallelism of management and information processes.

Section 2 provides an overview of the current state of research in the subject area under consideration. Section 3 presents the methodology developed by the authors for solving the problems of formal description of the problems of describing natural parallelism. The formal description of the problem under consideration is carried out. Section 4. describes the obtained results and proposed approach advantages. Section 5 presents possible applications of the obtained results and directions for further research.

2. State of the art

There are many variants of formal description of natural parallelism of processes occurring in CO and ACS. Among the variants are methods based on the introduction of logic functions (Zhuk et al., 2014; Vasiliev, 1999), step functions (Moiseev, 1981; Zimin & Ivanilov, 1971), phase and mixed constraints given in the form of equalities and inequalities (Kalinin & Sokolov, 1995). But the mentioned methods and corresponding models aren't devoid of disadvantages (the criticality to the dimensionality of the tasks to be solved, the difficulties of taking into account a number of constraints, which include limitations on the discontinuity of interaction operations (IO) CO) that hinder their widespread use in practice. The articles (B. Sokolov et al., 2020; Boris Sokolov et al., 2018) propose a new approach to the formalization of conjugate representation based on logical-dynamic constraints implemented using mixed constraints. The simple form of such formalization could be represented as the following logic and dynamic model of program control IO CO:

$$\Delta = \left\{ \boldsymbol{u} | \dot{x}_i = \sum_{j=1}^{m} u_{ij} ; \sum_{i=1}^{n} u_{ij}(t) \le 1; \sum_{j=1}^{m} u_{ij} \le 1; \right.$$

$$u_{ij}(t) \in \{0,1\};\ t \in (t_0, t_f] = T;\ \ x_i(t_0) = 0;$$

$$x_i(t_f) = a_i; \qquad (1)$$

$$\sum_{j=1}^{m} u_{ij} \left[\sum_{\alpha \in \Gamma_{1i}^-} (a_\alpha - x_\alpha(t)) + \prod_{\beta \in \Gamma_{i2}^-} \left(a_\beta - x_\beta(t) \right) \right] = 0;$$

$$i = 1, \dots, n;\ j = 1, \dots, m$$

where $x_i(t)$ – a variable characterizing the state of execution of the interaction operation D_i, at time t; $u_{ij}(t)$ – control action, taking value 1, if the interaction operation D_i is performed using B_j-resource ("subsystem") of the ACS CO, 0 - in the opposite case; a_i – the given volume of performance of the specified operation; $\alpha \in \Gamma_{1i}^-$, $\beta \in \Gamma_{2i}^-$ – the set of numbers of operations immediately preceding and technologically related to the operation D_i by means of the logical operations "AND", "OR" ("XOR"); T – time interval of CO ACS operation; t_0, t_f – initial and final time moments.

In accordance with the work (B Sokolov et al., 2022; B. V. Sokolov et al., 2021), the use of the model (1) describes at the same time operational and flow processes, parallel-sequentially executed in modern ACS CO, by different interpretation of the components of the state vector of the interaction operation D_i and the vector of control actions._The works (Cao et al., 2009; Marik et al., 2020; Peterson, 1977) present other variants of modern approaches to modeling parallel-sequential distributed asynchronous processes of the form (1) used in the research of COs and their control systems.

3. Petri Net Dynamic Interpretation

As an example of a discrete description of a model of the form (1), let us consider a simplified version of the dynamic interpretation of the Petri net using a discrete dynamic system (DDS) (Peterson, 1977). These networks allow a formal representation of parallel-sequential various types of asynchronous processes (as management and information) occurring in the CO ACS.

The originality of the proposed description is that it has a system-management interpretation, allowing (in contrast to traditional approaches) already correctly formulate and solve a wide range of managerial problems using Petri nets. In this interpretation of the Petri net, it is assumed that the value of each i-th component of the state vector DDS of the form: $\mathbf{x}[l] = ||x_1[l], x_2[l],\dots, x_n[l]||^{T}$, $l = 1,\dots,N$ (l – current step index, point in time). It equals to the total number of labels in p_i positions in the initial Petri net, and each t_j transition is matched with a control action $u_j[l] \in \{0,1\}$, taking the value 1 if t_j transition is triggered at step l, 0 otherwise. In addition, at triggering of each allowed transition the movement of labels from one position to another is not instantaneous, but with a fixed duration (step). In this case, the equations describing dynamics of change of labels in the considered Petri net can be set in the form of the following recurrence relations:

$$x_i[l] = x_i[l-1] + \textstyle\sum_{\beta \in \Gamma_i^-} k_\beta u_\beta[l] - \sum_{\alpha \in \Gamma_i^+} k_\alpha u_\alpha[l], \qquad (2)$$

where k_β, k_α – the multiplicity of edges connecting respectively t_j transitions with p_i position and p_i position with t_j transitions;

Γ_i^- (Γ_i^+) – set of numbers of input (output) transitions p_i position. In addition to (2) it is necessary to set the constraints in the DDS describing the Petri network structure and the logic of switching transitions. These constraints can be represented as follows:

$$u_\alpha[l] \textstyle\sum_{i \in I_\alpha} \prod_{\xi=k_i}^{s_i} (\xi - x_i[l-1]) = 0, \qquad (3)$$

$$\textstyle\sum_{\alpha \in \Gamma_i^+} k_\alpha u_\alpha[l] \le x_i[l-1], \qquad (4)$$

$$u_\alpha[l] \textstyle\sum_{\nu \in J_\alpha} x_\nu[l-1] = 0, \qquad (5)$$

where $s_i = \max x_i[l]$, $l = 1,\dots,N$ –the maximum possible number of labels that can be in position p_i; $I(J\alpha)$ – set of numbers of input positions (output positions with constraining edges) for t_j transition. Together with (3)-(5), the initial and final (required) labeling of the Petri net x[0], x[N]; the quality index of complex objects functioning: $F=\sum_{l=1}^{N} g_l(\boldsymbol{x}[l-1],\boldsymbol{u}[l])$, where $g_l(\cdot,\cdot)$ – given functions.

Note that the proposed approach to the formalization of the CO functioning processes is applicable in those cases where the mentioned processes are described by multicolored temporal Petri nets. The main peculiarity of the proposed dynamic interpretation of Petri nets is to set relations (2) - (5) in such a way that the integrability of the state vector components and controls in the constructed DDS at each step l = 1,..., N.is ensured.

4. Results and Discussion

The proposed logic and dynamic description (LDD) of Petri nets allows one to significantly extend the capabilities of this mathematical apparatus by involving the fundamental and applied results obtained in classical and modern control theories (Bellman & Kalaba, 1965; Boltyanskii et al., 1960; Chernous' ko, 2018; Chernous' ko et al., 2008; Ivanov et al., 2021; Васильев, 1999). Let us explain this situation by the example of construction and use of approximated reachability area (RA) for LDD of Petri nets of the form (2). The problem of constructing RA for models of the form (2) is equivalent in its content to the problem of constructing reachability trees (RT) for classical Petri nets. But in the latter case, this problem belongs to the class of NP problems in terms of computational labor intensity. However, if this problem is reformulated as an optimal control problem, it can already be solved in finite time with the required accuracy (Kalinin & Sokolov, 1995) .

One of the possible methods (algorithms) of constructing the RA of the Petri network, which is based on the multiple solution of the optimal program control of recurrent dynamic system of the form (2) with a functional of the form (Chernous' ko et al., 2008; Chernousko, 1988; Boris Sokolov et al., 2018):

$$J''_{gen}(\boldsymbol{x}(\cdot))=\boldsymbol{c}^m\boldsymbol{x}(N)\to\min_{\boldsymbol{u}\in Q_p(\boldsymbol{x})},\qquad(6)$$

where $\boldsymbol{c}$ – a given vector satisfying the normalization conditions $|c|=1$. We search $\boldsymbol{u}^*(l)$ for each fixed c in order to obtain a point $\boldsymbol{x}^*(N)$ on the boundary of the reachability set and a reference hyperplane of the form $\boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}^*(N)$ to this set passing through the point $\boldsymbol{x}^*(N)$. Let us define $\boldsymbol{x}^*_{\beta}(N)$ and the reference hyperplanes for the given variants of variation components of the vector $\boldsymbol{c}_{\bar{\beta}}$, $\bar{\beta}=1,\ldots,\bar{\bar{\Delta}}$ ($\bar{\bar{\Delta}}$ - the number of variants) to obtain an external ($D^{(+)}$), and internal ($D^{(-)}$) approximation of the reachability set $D(N,1,\boldsymbol{x}(1))$.

In the work (B. V. Sokolov et al., 2021; Ushakov, 2020) it was shown that in the general case for the logical-dynamic description of Petri net (2) of the class under consideration the external approximation $D^{+}(N,1,\boldsymbol{x}(1))$ of the set $D(N,1,\boldsymbol{x}(1))$ will be a convex polyhedron formed by the intersection of reference hyperplanes. An internal approximation $D^{-}(N,1,\boldsymbol{x}(1))$ of the set $D(N,1,\boldsymbol{x}(1))$ can be a convex polyhedron whose vertices are points $\boldsymbol{x}^*_{\beta}(N\ )$ or differently$D^{-}(N,1,\boldsymbol{x}(1))=\mathrm{Co}(\boldsymbol{x}_1(N),\ldots,\boldsymbol{x}_{\bar{\bar{\Delta}}}(N))$. The more $\bar{\bar{\Delta}}$, the better D^+ and D^- approximate the reachability set. In the considered case (Okhtilev et al., 2006) the value $\bar{\bar{\Delta}}$ is determined by the total number of possible interruptions of interaction operations in CO at a given time interval $(1,l)$.

The basis of the algorithm for constructing the polyhedron D^+ and D^- is sequential approximation method(Chernousko, 1988). Instead of varying the values of the vector components c in the functional (6), one should vary the values of the vector components $\psi(1)$, which represents the vector of conjugate variables at the initial moment of time. In this case, the solution of the initial complex boundary value problem is replaced by the solution of Cauchy problems for differential equations (2). The advantage of the proposed approach consists in the fact that in the proactive control of CO the components of the vector $\psi(T)$ have a certain meaningful interpretation (Ivanov et al., 2021), which allows one to simplify the procedure of enumerating the values of the indicated vector and reduce the total amount of calculations.

5. Conclusions

At present, based on the proposed dynamic description of the natural parallelism of IP and management processes in ACS CO in continuous and discrete forms, it has been possible to solve a number of important scientific and applied problems (Kalinin & Sokolov, 1995; Okhtilev et al., 2006; ; Zakharov, 2022). The developed algorithms for finding optimal schedules of means functioning can be used to find optimal rules for triggering transitions in Petri nets by evaluating the achievability of a given labeling(Gorodetsky & Kotenko, 2004). On the other hand, when solving various problems of the theory of schedules and modeling and simulation, problems of structural and functional synthesis of CO, using the mathematical apparatus of Petri nets, one can carry out a constructive evaluation of the time and capacitance complexity of corresponding optimization algorithms, the search for dispatch schedules (first approximations) in the problems of optimal program control of complex operations (Ivanov et al., 2022; Zakharov, 2022). In addition, the proposed set of dynamic models can be used to evaluate and select the best system modeling techniques for a given class of CO , which could not be done earlier.

At the same time, the approach has a number of limitations related to computational complexity, so, for example, it cannot be used to solve problems in real time. This is due to the need to take into account interruptions of operations and their analysis. This is largely due to the ratio of resources used and the regulatory volumes of the relevant processes.

A further area of research is the development of a discrete-event simulation model for the synthesis of hardware and software characteristics with a fixed schedule in modern information and computing networks (industrial Internet) using methods of mathematical programming and stochastic optimization, which will allow taking into account not only complex spatial-temporal, technical and technological constraints, but also the effects of disturbing factors on complex object.

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