# A flow shop scheduling approach for the productionrouting problem with complex setups constraints 

Juan Pablo Cisneros ${ }^{1 *}$, Idalia Flores ${ }^{1}$<br>${ }^{1}$ National Autonomous University of Mexico (UNAM), Av. Universidad 3000 Copilco Universidad, Mexico City, 04510, Mexico<br>*Corresponding author. Email address: jupablo.cisneros@gmail.com


#### Abstract

In a Vendor Managed Inventory (VMI) supply chain there is the challenge to deliver the products to the consumers within desired time at minimum cost. This paper proposes an optimization model to solve the production-routing problem (PRP) minimizing both manufacturing and transportation costs; this model excels in the consumer-packaged goods industry with flexible, interconnected, and complex manufacturing networks. This work's contribution relies in the optimization model approach by having the variables in units, unlike jobs, which has been the standard in literature but not in the industry. Also, this paper offers a flow shop scheduling scheme for manufacturing processes containing set ups constraints with a from-to products matrix behavior.


Keywords: production-routing problem; supply chain optimization; flow shop scheduling; manufacturing network; setups

## 1. Introduction

Throughout the years, companies have had an arduous search to position their consumer goods within the reach of the general population. With the evolution of modern manufacturing came the establishment and management of supply chains (Mourtzis and Doukas, 2014). Usually, the supply chain begins with the supply of raw materials, to be then processed at manufacturing facilities, where the raw material is transformed, and a finished product is obtained. Later, this finished product is taken to warehouses where transportation routes are created to finally deliver the requested products to customer facilities.

Costs are incurred in each of the steps mentioned, which can be grouped into production, storage, and transportation costs. Once the supply chain has been successfully managed, the next logical step in decision making is to find room for improvement in
optimization to minimize the total landed cost.
The structure for this work state as follows, in section 2 , the state of the art for the productionrouting problem is addressed. Then, in section 3 the optimization model proposal is presented. After, in section 4 a case study to validate the model is offered followed by the results and discussion in section 5 . Finally, in section 6 the conclusions and next research lines are discussed.

## 2. State of the art

Given a network where products flow, the possibility of doing so at a lower cost has been approached for decades. The first time this problem was documented was as a generalization of the classic Traveling Salesperson Problem (TSP) (Dantzig and Ramser, 1959) to originate what later became the Vehicle Routing Problem (VRP) that has continued to be developed along numerous research lines in recent
decades. A generalization of VRP came through the inclusion of inventory management, resulting in the Inventory Routing Problem (IRP), first mentioned in a daily delivery planning application for the Air Products \& Chemicals company (Bell et al., 1983), where they generated operating savings from $6 \%$ to $10 \%$. The IRP problem was later formulated and generalized (Federgruen and Zipkin, 1984).

At this point in time, no author had integrated manufacturing within the model, but it is essential to include this section to optimize the entire supply chain. Thus, the production-routing problem (PRP) arises from the need to optimize not only the routing and storage of products, but also the production phase. Consequently, the PRP is created by joining two historical models, the routing and inventory part comes from the IRP and it is mixed with the production facet of the Lot Sizing with Direct Shipment (LSDS) (Adulyasak et al., 2015).

The first PRP model aimed to minimize both fixed setup production costs and distribution costs in an integral model of production and distribution (Chandra and Fisher, 1994). With this article as a starting point for the PRP, they only considered a single manufacturing plant with fixed costs. Specifically, a minimization of the variable production cost is not included in their objective function since there are no costs variations that usually is obtained when producing at various facilities. The first model that considers a multi-plant optimization can be seen in the work of (Lei et al., 2006). From these contributions, an important development in the resolution of PRPs began, which has had a boom in the last decade, an extensive review of literature and resolution algorithms has been developed already (Adulyasaak et al., 2015).

The PRP has been subject to numerous applications. Such is the case of (Neves-Moreira et al, 2019), where a problem with several production lines and deliveries with time windows is addressed. In (Qiu et al., 2021) they solve a two-echelon supply chain problem by having satellite cross-docks between the plant and the customers. Alternatively, not all applications seek financial benefits, in (Qui et al., 2017) they propose a branch-and-price algorithm to minimize CO 2 emissions through the resolution of the PRP. While our work presents a specific case study, we believe that the proposed formulation could be applied in several industries that mainly are established as Vendor Managed Inventory (VMI) supply chains where vertical integration is a main factor.

Furthermore, some authors have solved the jobshop scheduling problem with simulation tools (Bottani et al., 2017), but since this type of facility with department-oriented process layout pretends to solve low volume and high flexibility industries, this approach focuses more on make-to-order (MTO) businesses. On the other hand, flow-shop oriented facilities are more suited for high volume and low
flexibility by having a product layout which is more aligned with VMI supply chains for consumerpackaged goods industry (Zhao, 2019) which is addressed in this work.

Accordingly, the flow-shop scheduling problem solves the order in which products should be processed in a factory that is normally designed with successive processes and parallel machines where product flows in a single consecutive and continuous direction. Within this approach, some authors have formulated to minimize the total time starting at manufacturing to final delivery (Yagmur and Kesen, 2021), whereas others have continued to be aligned with cost minimization for the entire supply chain (ScholzReiter et al., 2011). In this work, we mainly overlap with the latter case, however, the main difference and one of the key contributions is the definition of the model unit. The traditional flow shop scheduling is based on jobs, each job must be previously defined as the sum or the total amount of product to be manufactured and transported for a single order, yet companies typically manufacture in high volumes and consolidate orders to be delivered only at warehouses by using a specific unit through the entire supply chain such as pieces, boxes, kilograms, liters, etc. In consequence, the model presented in the following section is based on mere units to have enough flexibility to be implemented in several cases, removing the ambiguity caused by jobs that need to be predefined.

As mentioned previously, the PRP started by focusing on setups, in this case (Chandra and Fisher, 1994), in the minimization of the setup costs. In contrast, the flow-shop scheduling problem minimizes the setup times between different products. Alternatively, something that has not been explored in the literature, is the special case of setup times depending on from-to matrix behavior. This type of behavior is commonly found in the food industry, where different SKUs are made within the same production lines and setup duration times depend on from which type of product the change is taking place to the type of product that runs next.

Since this problem contains the VRP which is NPhard, hence, the resolution of the PRP model to be proposed will be as well NP-hard. For this reason, the computational complexity problem also arises. To solve this, some authors have developed heuristics (Avci and Yildiz, 2020; Meinecke and Scholtz-Reiter, 2014; Qiu et al., 2017), while others have opted for metaheuristic techniques (Neves-Moreira and Almada-Lobo, 2019; Qiu et al., 2018). Nevertheless, this paper does not focus on the resolution methods for the PRP or the flow-shop scheduling. Our contributions obey entirely to the novelties presented in the model formulation, reflected in the units instead of jobs case and setups with from-to behavior.

## 3. Problem formulation

The mathematical model was developed for a multifacility flexible network where the same product can be processed in different factories. In this model, a single echelon is considered, starting in factories, and carrying the product to the company's warehouses, where the demand is placed. This type of supply chain management is typically implemented by the consumer-packaged products industry which often operate in a VMI scheme. Since there is no intention to have finished product storage within the plants, no inventories will be considered for this model, therefore, the model will only consider production and transportation costs.

Moreover, this model pretends to solve the scheduling for the manufacturing phase. Also, this formulation solves complex setup times constraints, which behavior can be explained by a for-to matrix. As mentioned previously, this constraint is common in the food industry due to allergens, different meat species or any factor that involves a deep cleaning, e.g., in an ice cream factory, vanilla and chocolate flavor products are processed on the same line, the cleaning time between flavors from vanilla to chocolate could be half an hour, but from chocolate to vanilla the cleaning time could be increased to one hour.

The complete optimization model is described below.

### 3.1. Sets

## Products $j \in J$

Machines $m \in M$
Production level $n \in N$
Production order $o \in O$
Factory $f \in F$
Warehouse $a \in A$

### 3.2. Parameters

$c_{j f}^{p}$ : Unitary production cost of product $j$ at factory $f$
$c_{f a}^{t}$ : Unitary transportation cost from factory $f$ to warehouse a
$t^{p}{ }_{j m n f}$ : Unitary production time of product $j$ at machine $m$ at production level $n$ at factory $f$
$t^{a}{ }_{j j \text { jimnf }}$ : Setup time from product $j$ to $j^{\prime}$ at machine $m$ at level $n$ at factory $f$
$H D_{m n f}$ : Available hours per machine $m$ at level $n$ at factory $f$
$D_{j a}$ : Demand for product $j$ at warehouse a
M: very large number
$x_{\text {jmnof }}^{p}$ : production units for product $j$ at machine $m$ at level $n$ at order o at factory $f$
$x_{j f a}^{t}$ : transportation units for product $p$ from factory $f$ to warehouse a
$t^{f}{ }_{j m n f}$ : completion time for product $j$ at machine $m$ at level $n$ at factory $f$

### 3.4. Binary variables

$X_{\text {jmnof }}$ : binary variable denoting that product $j$ is processed at machine $m$ at level $n$ at order $o$ at factory $f$
$Y_{j j \not m n f}$ : binary variable denoting that product $j$ is processed before product $j$ ' at machine $m$ at level $n$ at factory $f$

### 3.5. Objective Function

The objective function minimizes the total landed cost given by the sum of production and transportation costs.

$$
Z_{\text {min }}=\sum_{j=1}^{J} \sum_{m=1}^{M} \sum_{n=1}^{1} \sum_{f=1}^{F} c_{j m n f}^{p} * x_{j m n o f}^{p}+\sum_{j=1}^{J} \sum_{f=1}^{F} \sum_{a=1}^{A} c_{f a}^{t} * x_{j f a}^{t}
$$

(1)

### 3.6. Constraints

The amount of product that is transferred from the factories must satisfy the demand assigned to each warehouse.

$$
\sum_{f=1}^{F} x_{j f a}^{t} \geq D_{j a}(\forall a, j)
$$

(2)

This model does not contemplate the capacity to store inventories, so the amount of product that is manufactured must be equal to the amount that is transported.

$$
\begin{equation*}
\sum_{m=1}^{M} \sum_{n=1}^{1} \sum_{o=1}^{o} x_{j m n o f}^{p}-x_{j f a}^{t}=0(\forall j, f, a) \tag{3}
\end{equation*}
$$

The completion time for every product must be less than the available hours that each machine $m$ in every level $n$ has.

$$
\begin{equation*}
t^{f}{ }_{j m n f} \leq H D_{n m f}(\forall j, m, n, f) \tag{4}
\end{equation*}
$$

The binary variable $X_{j m n o f}$ is being activated when at least one unit of $x_{\text {jmnof }}^{p}$ is processed, these binary variables are necessary to trigger setups in the following constraint.

$$
\begin{equation*}
M X_{\text {jmnof }} \geq x_{\text {jmnof }}^{p}(\forall j, m, n, o, f) \tag{5}
\end{equation*}
$$

### 3.3. Positive variables

The binary variable $Y_{j j^{\prime} \text { mnof }}$ is conditioned to activate only if the binary variables $X_{\text {jmnof }}$ and $X_{j^{\prime} m n o+1 f}$ take the value of 1 , therefore, there will only be a setup between $j$ and $j^{\prime}$ when they correspond to the order $o$ y $o+1$, respectively.

$$
\begin{equation*}
1-M\left(2-X_{j m n o f}-X_{j^{\prime} m n o+1 f}\right) \leq Y_{j j^{\prime} m n f}(\forall j, m, n, o, f) \tag{6}
\end{equation*}
$$

For every machine $m$ in every level $n$ only one product can be processed at a time, in this way the sequence for each order $o$ is respected. This constraint will only be active when the product $X_{j m n o f}$ is followed by $X_{j^{\prime} m n o+1 f}$, likewise, only in this case the setup $Y_{j j^{\prime} m n f}$ is activated.

$$
\begin{array}{rl}
t_{j m n f}^{f}+t^{p}{ }_{j^{\prime} \text { mnf }} * & * x_{j^{\prime} \text { mno }+1 f}^{p}+t^{a_{j j^{\prime} m n f} * Y_{j j^{\prime} m n f}} \\
& \leq t^{f^{\prime} j^{\prime} m n f}+M\left(2-X_{j m n o f}-X_{j^{\prime} m n o+1 f}\right) \\
& +M\left(1-Y_{j j^{\prime} m^{\prime} m f}\right)(\forall j, m, n, o, f) \tag{7}
\end{array}
$$

For the same product $j$, its completion times are forced to go up as it progresses through levels $n$, consequently, it cannot enter level 2 if it has not finished processing in level 1.

$$
\begin{array}{rl}
t^{f}{ }_{j m n-1 f}+t^{p}{ }_{j m n f} & * x_{j m n o f}^{p}+t^{a} j_{j^{\prime} \text { jmnf }} * Y_{j^{\prime}{ }_{j m n f}} \\
& \leq t^{f}{ }_{j m n f}+M\left(1-X_{j m n o f}\right)(\forall j, m, n, o, f) \tag{8}
\end{array}
$$

For a product to be considered as finished, all units must be processed through all production levels.

$$
\begin{equation*}
\sum_{o=1}^{o} \sum_{m=1}^{M} x_{j m n o f}^{p}-\sum_{o=1}^{o} \sum_{m=1}^{M} x_{j m n+1 o f}^{p}=0(\forall j, n, f) \tag{9}
\end{equation*}
$$

For each machine $m$ at every level $n$, only one product $j$ can be considered for a single order $o$. Likewise, a product can only be processed once, that is, in a single order $o$.

$$
\begin{equation*}
\sum_{j=1}^{J} X_{j m n o f} \leq 1(\forall m, n, o, f) \tag{10}
\end{equation*}
$$

Continuous decision variables cannot take negative values.

$$
\begin{equation*}
x_{j m n o f}^{p}, x_{j f a}^{t}, t_{j m n f}^{f} \geq 0 \tag{11}
\end{equation*}
$$

Binary decision variables can only take the values zero or one.

$$
X_{j m n o f}, Y_{j j \text { jmnf }}=\{0,1\}
$$

## 4. Case study

To validate the mathematical model formulation a small instance of the problem was created and solved in Lingo 19.0 software. This instance was specifically developed to challenge the entire formulation. To do so, the previously described ice cream factory case is extended.

The parameters were designed to find a solution for a factory with limited capacity, hence, the model obtained a solution that fulfilled the time constraints by manufacturing the entire demand at a minimum cost.

In order to simplify case of study, we will consider a single factory and warehouse supply chain. In this factory, one-liter chocolate and vanilla flavored ice cream are produced and transported to the central warehouse. The ice cream manufacturing process is exemplified in two processes: the first stage or production level, represents the mixing and freezing, and in the second level the ice cream is packaged. One machine per level is considered, therefore, both flavors of ice cream must be processed in these two machines. Given this problem statement, the sets are defined below:

Products: J=\{1,2\} (1 = chocolate 2 = vanilla)
Machines: $\mathrm{M}=\{1\}$ (Although there are two machines, there is actually only one machine)

Production level: $\mathrm{N}=\{1,2\}$ ( $1=$ mixing and freezing, 2 = packaging)

Production order: $\mathrm{O}=\{1,2\}$ (consecutive processing order per machine)

Factories: $\mathrm{F}=\{1\}$ (single factory)
Warehouses: $\mathrm{A}=\{1\}$ (single and central warehouse)

The model parameters state as follows, a week of production with 150 available hours are considered for each machine and there is a demand for 95 liters of chocolate and 90 liters of vanilla. The unit production costs are 6 and 5 monetary units (M.U.), respectively; Additionally, the cost of transportation from the plant to the warehouse is 3 M.U. per liter. Also, the mixing and freezing processing times are 15 minutes for the chocolate flavor and 1 hour for vanilla; and for the packaging process the process time is 1 hour for chocolate and 15 minutes vanilla.

In addition, due to the food coloring, the cleaning time in the mixing and freezing line is 10 hours if it the change is made from chocolate to vanilla, but if it is done from vanilla to chocolate, only 5 hours are needed since it is easier to clean the coloring found in vanilla ice cream. On the other hand, due to the frozen product consistency, on the packaging line the cleaning times considered are 2 hours from chocolate to vanilla and 3 hours otherwise.

Since the number of constraints increases combinatorically, with the sets defined, a total of 55 constraints, 16 binary variables and 30 total variables were obtained. The problem was solved with the Lingo 19.0 optimizer. The solution was obtained in 0.1 seconds after 65 iterations, in a computer with 16 GB RAM memory and Intel ${ }^{\circledR}$ Core ${ }^{\mathrm{TM}}$ i7 processor.

## 5. Results and Discussion

### 5.1. Model verification: Extreme conditions test

The verification stage looks for the model to be conceptually correct. To do so, the conceptual model is established first, which it can be seen in Figure 1, The central box represents the main internal factors and dependencies that affect the output of the model, such as the parameters, objective function, and constraints. This central box is impacted by the external factors on the left box, the most relevant being the demand which is influenced by the market. Finally, the right box contains the expected results obtained by the model, in this case, the expected results is the total landed cost, production scheduling and routing.


Figure 1. Conceptual model for the proposed formulation
The conceptual model delimits the formulation scope and creates the baseline to start a simulation study. For this work, the simulation is not expanded, yet the conceptual model creates the opportunity to analyze the dependencies amongst model elements to establish the verification process of the model formulation.

To verify the model, the extreme conditions test was developed. This test consists in taking one or more parameters of the model and change their values to extreme conditions, that is, give the parameter a minimum value (zero) as well as a maximum value (infinity).

Then, in the general verification the objective function is evaluated to verify if the obtained value is an expected result. If a drill down in the particularities of the model, such as restrictions, is needed; the partial verification takes place by going one step deeper in the model dependencies. In both verifications the objective is to understand if there are atypical behaviors that do not make sense, so that a correction in the model is made.

The selected parameters to perform the extreme conditions test are production costs, demand, and processing times. These variables will tend to a maximum value of infinity and to a minimum value of zero. The results can be seen in Table 1. Extreme conditions test results.

Table 1. Extreme conditions test results.
\(\left.\left.$$
\begin{array}{cccl}\hline \text { Parameter } & \begin{array}{l}\text { Extreme } \\
\text { condition } \\
\text { value }\end{array} & \begin{array}{l}\text { Objective } \\
\text { function } \\
\text { value }\end{array} & \text { Comments } \\
c_{j f}^{p} & \infty & \infty & \begin{array}{l}\text { The cost maximizes } \\
c_{j f}^{p}\end{array} \\
D_{j a} & 0 & 0 & \begin{array}{l}\text { Only transportation } \\
\text { component remains in cost } \\
\text { Completion times tend to } \\
\text { infinity when they should be }\end{array} \\
\text { lower or equal than machine } \\
\text { available hours }\end{array}
$$\right] \begin{array}{l}Nothing happens and the <br>

model in interrupted, in a\end{array}\right]\)| partial manner, the |
| :--- |
| production queues are empty, |
| and the machines are idle |
| Completion times tend to |
| infinity when they should be |
| lower or equal than machine |
| available hours |
| A cost bound is obtained but |
| the model leaves the initial |
| $t_{j m n f}^{p}$ |

Having successfully done the extreme conditions test, the first five cases show predictable and coherent behavior. It is only in the last case, when the processing times tend to zero, that the verification phase does not gives us enough information, thus, the validation is the next step in the modelling process.

### 5.2. Model validation: Case study results

The model validation is done through the resolution of the case study mentioned previously. The optimal solution's objective function returns the value of the sum of the costs which adds to 1,575 M.U. For this instance, since there is only a single production and transportation cost by product and no back orders are allowed, if a feasible solution is obtained it will be optimal under the parameters described. To obtain a feasible solution, each constraint must be satisfied, the first constraint seen in equation (2) guarantees that the demand is fulfilled, values in Table 2 prove this.
Table 2. Solution for production units variables $x_{\text {jmnof }}^{p}$.

| j | m | n | o | f | Value |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 1 | 1 | 1 | 1 | 95 |
| 2 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 2 | 1 | 0 |
| 2 | 1 | 1 | 2 | 1 | 90 |
| 1 | 1 | 2 | 1 | 1 | 95 |
| 2 | 1 | 2 | 1 | 1 | 0 |
| 1 | 1 | 2 | 2 | 1 | 0 |
| 2 | 1 | 2 | 2 | 1 | 90 |

The prosed formulation pretends that by having different manufacturing costs through $F$ factories, as
well as numerous transportation cost depending on the combination between destinations, an optimal supply chain plan is obtained which chooses the optimal location to manufacture each product, in addition to routing. Nevertheless, although the objective function does not consider completion time minimization, in order to be able to manufacture the entire demand, the obtained solution naturally minimized the completion time due to the limited available hours capacity for the proposed factory.

Moreover, the obtained optimal solution contemplates starting to produce product 1 (chocolate) before product 2 (vanilla) as seen in Table 2. Each product follows the production flow so that the same units that are processed in level $n=1$ are also processed in level $n=2$. Then notice that for each level a single order is assigned to a product. These results comply with the constraint declared in equation (9).

As declared in equation (3), all manufactured products are transported to the central warehouse $a=$ 1, this can be seen in Table 3.

Table 3. Solution for transportation units variables $x_{j f a}^{t}$.

| $j$ | $f$ | $a$ | Value |
| :--- | :--- | :--- | :--- |
| 1 | 1 | 1 | 95 |
| 2 | 1 | 1 | 90 |

In Table 4, the value obtained for completion times are lower or equal than the 150 available hours that each machine has, complying with equation (4). Although the cleaning time is more extensive when starting to manufacture product 1 (chocolate) than when producing product 2 (vanilla) first; the optimal and only feasible solution is found when the total production time is minimized, this occurs by the flow shop scheduling modelling approach because of the shorter unit production time of product 1 at level 1.

| Table 4. Solution for completion times variables $t^{f}{ }_{j m n f}$. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| j | m | n | f | Value |
| 1 | 1 | 1 | 1 | 23.75 |
| 2 | 1 | 1 | 1 | 125.5 |
| 1 | 1 | 2 | 1 | 125.5 |
| 2 | 1 | 2 | 1 | 150.0 |

Furthermore, values described for the production binary variables in Table 5 describe that no production order is repeated for more than 1 product for each level, therefore, this guarantees that equation (10) is satisfied.

Table 5. Solution for production binary variables $X_{j m n o f}$.

| j | m | n | o | f | Value |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 2 | 1 | 0 |
| 2 | 1 | 1 | 2 | 1 | 1 |
| 1 | 1 | 2 | 1 | 1 | 1 |
| 2 | 1 | 2 | 1 | 1 | 0 |
| 1 | 1 | 2 | 2 | 1 | 0 |
| 2 | 1 | 2 | 2 | 1 | 1 |

Finally, the relationships established in equations (5), (6), (7) and (8) between decision variables mentioned previously in addition with values from Table 6 guarantee that the combinations for those constraints groups fulfill the flow-shop scheduling approach.

| Table 6. Solution for production binary variables $Y_{j j \not m n f}$. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :--- |
| j | $\mathrm{j}^{\prime}$ | m | n | f | Value |
| 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 2 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 1 | 1 | 0 |
| 2 | 2 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 2 | 1 | 0 |
| 1 | 2 | 1 | 2 | 1 | 1 |
| 2 | 1 | 1 | 2 | 1 | 0 |
| 2 | 2 | 1 | 2 | 1 | 0 |

In conclusion, the case study validates that the formulation correctly abstracts the reality, that is, the values obtained by the software are logical and correspond to what should be obtained in an analytical practice. In consequence, with the extreme conditions test in addition to the ice cream factory case study, the optimization model proposed for the productionrouting problem is both verified and validated. Also, the formulation solves the manufacturing sequencing with a flow-shop scheduling approach.

## 6. Conclusions

This paper successfully proposes a novel formulation to minimize the total landed cost for a supply chain with flow shop scheduling based on units with setups explained by a for-to behavior.

The case study validates the presented formulation, nevertheless, the proposed model rapidly increases in magnitude, therefore, the solving method must be addressed with other techniques or mathematical tools such as heuristics or metaheuristics to face more complex problems moving forward.

Notably, this first approach gives more importance to manufacturing than the transportation phase in a supply chain, for future work the objective is to also prioritize distribution formulation to have a fully detailed production-routing problem.

Finally, the next logical step for the proposed mathematical model is to migrate towards a simulation solution from the conceptual model mentioned to solve the manufacturing scheduling and distribution routing of a supply chain, this will help to address the problem in a more dynamic way enabling us to control time within the simulation.

## Funding

We specially thank the General Coordination of Postgraduate Studies for their support with the UNAM's scholarship for Postgraduate studies that aided the realization of this work.

## References

Adulyasak Y., Cordeau, J. F., Jans, R. (2015). The production routing problem: A review of formulations and solution algorithms. Computers \& Operations Research 55, 141-152.
Avci, M., Yildiz, S. T. (2020). A mathematical programming-based heuristic for the production routing problem with transshipments. Computers and Operations Research 123, 105042.
Bell, W. J., Dalberto, L. M., Fisher, M. L., Greenfield, A. J., Jaikumar, R., Kedia, P., Mack, R. G., Prutzman, P. J. (1983). Improving the Distribution of Industrial Gases with an On-Line Computerized Routing and Scheduling Optimizer. Interface, Dec., 1983, Vol. 13, No. 6, CPMS/TIMS Prize Papers, pp. 4-23.
Bottani, E., Rinaldi, M., Montanari, R., Bertolini, M., Zammori, F. (2017). A simulation tool for modelling and optimization of a job-shop production system. Proceedings of the European Modeling and Simulation Symposium, 2017 ISBN 978-88-97999-85-0; Affenzeller, Bruzzone, Jiménez, Longo and Piera Eds, 489-495.
Chandra, P., Fisher, M. (1994). Coordination of production and distribution planning. Eur J Oper Res, 72(3):503-17.
Dantzig, G. B. and Ramser, J. H. (1959). The Truck Dispatching Problem. Management Science, Vol. 6, No. 1, pp. 80-91.
Federgruenand, A., Zipkin, P. (1984). A Combined Vehicle Routing and Inventory Allocation Problem. Operations Research Vol. 32, No. 5, SeptemberOctober 1984.
Lei, L., Liu, S., Ruszczynski, A., Park, S. (2006). On the integrated production, inventory, and distribution routing problem. IIE Trans, 38(11):955-70.
Meinecke, C., Scholz-Reiter, B. (2014). A heuristic for the integrated production and distribution scheduling problem. International Science Index, 8(2), 290-297.
Mourtzis, D. and Doukas, M. (2014). Design and planning of manufacturing networks for mass customisation and personalisation: Challenges and Outlook. Procedia CIRP 191-13.
Neves-Moreira, F., Almada-Lobo, B., Cordeau, J.F., Guimarães, L. (2019). Solving a large multi-product production-routing problem with delivery time windows. Omega 86, 154-172.
Qiu, Y., Qiao, J., Pardalos, P. M. (2017). A branch-andprice algorithm for production routing problems with carbon cap-and-trade. Omega 68, 49-61.
Qiu, Y., Wang, L., Xu, X., Fang, X., Pardalos, P. M. (2018). A variable neighborhood search heuristic algorithm for production routing problems. Applied Soft Computing 66 (2018) 311-318.

Qiu, Y., Zhou, D., Du, Y., Liu, J., Pardalos, P. M., Qiao, J. (2021). The two-echelon production routing problem with cross-docking satellites. Transportation Research Part E 147, 102210.
Scholz-Reiter, B., Schwindt, C., Makuschewitz, T., Frazzon, E. M. (2011). An approach for the integration of production scheduling and interfacility transportation within global supply chains. International Journal of Logistics Systems and Management, 10(2),158-179.
Yagmur, E., Kesen, S. E. (2021). Multi-trip heterogeneous vehicle routing problem coordinated with production scheduling: Memetic algorithm and simulated annealing approaches. Computers \& Industrial Engineering 161, 107649.
Zhao, R. (2019) A Review on Theoretical Development of Vendor-Managed Inventory in Supply Chain. American Journal of Industrial and Business Management, 9, 999-1010.

