

SIMULATION-BASED EVALUATION OF AUTOMATED TRADING STRATEGIES: A MANIFESTO FOR MODERN METHODS

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ABSTRACT

In many investment banks and major fund-management companies, automated "robot" trading systems now do work that 20 years ago would have required large numbers of human traders to perform: the rise of robot traders is a major success-story for artificial intelligence (AI) research. Although the technical details of currently profitable automated trading systems are closely guarded commercial secrets, the rise of robot trading can be traced back to a sequence of key AI research papers. Each of these key papers relied on minimal abstract simulation models of real financial markets: the simulators provide test-beds for trials in which the performance of different trading strategies could be evaluated and compared. Recent studies have revisited these seminal results, using more realistic simulations of contemporary financial markets, and have cast major doubts on core conclusions drawn in the original publications. Therefore, it seems reasonable to argue that present-day simulation methods are exposing significant problems in past research on automated trading. This position paper presents no new empirical results but instead presents a review of key past papers and an argument, a manifesto, for establishing a shared market-simulator test-bed that adequately reflects current real-world financial markets, for use in future evaluation and comparison of trading strategies.

Keywords: financial markets, automated trading, simulation methodology.

1. INTRODUCTION

In the past fifteen years most major financial markets around the world have undergone seismic shifts in the extent to which highly-paid (and, presumably, highly intelligent) human traders have been replaced by automated trading systems known variously as *algorithmic trading systems* or more simply as *robot traders*. In many investment banks and major fund-management companies, automated trading systems now do work that previously would have required tens or hundreds of human traders to perform: in this sense, the rise of robot traders is a major success-story for artificial intelligence (AI) research.

Although the technical details of profitable automated trading systems are closely guarded commercial secrets, the rise of robot trading can be traced back to a sequence of key publications by academic and industrial AI researchers, commencing in 1993 and continuing through to 2008. Each of these key papers relied (to varying extents) on simulation models of real financial markets: the market simulators provide test-beds for trials in which the performance of different trading

strategies could be evaluated, and emphasis was given to establishing which strategy out-performed (or "dominated") other strategies previously described in the published literature. All of these key studies involved minimal, abstract simulation models of real financial markets. Results from this sequence of papers are widely cited and have until very recently been essentially unquestioned. However, recent studies have revisited these results, extending the nature of the trials that the various strategies are subjected to, and exploring their responses in more realistic simulations of contemporary financial markets: this has cast major doubts on core conclusions drawn in the original publications.

The recent studies involve highly compute-intensive brute-force exhaustive simulation approaches, methods that arguably would have been prohibitively expensive to attempt when the original research was undertaken in the 1990's. Thus, it seems that modern-day simulation methods are exposing significant problems in past research. This position paper presents no new empirical results but instead presents an argument, a manifesto, for establishing a modernized methodology for evaluating trading agents. Specifically, it is proposed here: (1) that the simple abstract models of markets that were used previously should be replaced by a simulation and modelling approach that more accurately reflects the micro and macro structure of present-day financial markets and the traders that interact within them, markets in which *co-adaptive dynamics* are a major factor; (2) that researchers pay more attention to the combinatorics of rigorous evaluation – if tens or hundreds of millions of simulation trials are required to rigorously establish a result, we should not shy away from that; and (3) that open-source software methods should be fully exploited to ensure that the international community of researchers working on automated trading share common simulation tools, thereby easing replication and extension of earlier results. In the final section of this paper, I describe a major update to an existing open-source financial-market simulator which is now offered as a freely available resource for the research community. The updated simulator captures many aspects of current financial markets that have been absent in previous simulation-based studies, and is offered as a free resource to the community in the hope that it becomes a trusted common test-bed for future simulation-based evaluation of automated trading strategies.

In Section 2 we explain the background to this work: there is quite a lot to cover. Section 2.1 briefly reviews the rise of automated trading in the global financial markets. Section 2.2 then introduces the concepts and terminology from the economics of market

microstructure that are relevant to the discussion here. After that, Section 2.3 reviews a sequence of key papers in the field and then Section 2.4 discusses recent papers which, using contemporary simulation approaches, overturn conclusions drawn in the earlier papers. Sections 3 and 4 then respectively discuss microstructural and macrostructural issues in simulating financial markets for evaluating automated trading systems. This leads into Section 5's closing description of a modernized simulator (i.e. one that better reflects the structure of current financial markets) created by extending an established public-domain open-source exchange simulator, and its intended use-cases.

2. TRADERS, MARKETS, AND KEY PAPERS

The 2002 Nobel Prize in Economics was awarded to Vernon Smith, in recognition of Smith's work in establishing and thereafter growing the field of *Experimental Economics* (abbreviated hereafter to "EE"). Smith showed that the microeconomic behaviour of human traders interacting within the rules of some specified market, known technically as an *auction mechanism*, could be studied empirically, under controlled and repeatable laboratory conditions, rather than in the noisy messy confusing circumstances of real-world markets. The minimal laboratory studies could act as useful proxies for studying real-world markets of any type, but one particular auction mechanism has received the majority of attention: the *Continuous Double Auction* (CDA), in which any buyer can announce a bid-price at any time and any seller can announce an offer-price at any time, and in which at any time any trader in the market can accept an offer or bid from a counterparty, and thereby engage in a transaction. The CDA is the basis of major financial markets worldwide.

Each trader in one of Smith's experimental CDA markets would be assigned a private valuation, a secret *limit price*: for a buyer this was the price above which he or she should not pay when purchasing an item; for a seller this was the price below which he or she should not sell an item. These limit-price assignments model the client orders executed by sales traders in real financial markets; we'll refer to them just as *assignments* in the rest of this paper. Traders in EE experiments from Smith's onwards are often motivated by payment of some form of real-world reward that is proportional to the amount of *profit* that they accrue from their transactions: the profit is the difference between the limit price specified when a unit is assigned to a trader, and the actual transaction price for that unit.

The limit prices in the assignments defined the market's supply and demand schedules, which are commonly illustrated in economics texts as supply and demand curves on a 2D graph with quantity on the horizontal axis and price on the vertical axis: where the two curves intersect is the market's theoretical competitive equilibrium point – a pair of (price, quantity) coordinates. A fundamental observation from microeconomics (the study of markets and prices) is that competition among buyers pushes prices up, and

competition among sellers pushes prices down, and these two opposing influences on prices balance out at the competitive equilibrium point; a market in which transaction prices rapidly and stably settles to the theoretical equilibrium price is often viewed by economists as *efficient* (for a specific definition of efficiency) whereas a market in which transactions consistently occur at off-equilibrium prices is usually thought of as inefficient: for instance, if transaction prices are consistently above the theoretical equilibrium price then it's likely that buyers are being ripped off. By varying the prices in the assignments to the traders, the nature of the market's supply and demand curves could be altered, and the effects of those variations on the speed and stability of the market's convergence toward an equilibrium could be measured.

Smith's initial set of experiments were run in the late 1950's, and the results and associated discussion were presented in his first paper on EE, published in the highly prestigious *Journal of Political Economy* (JPE) in 1962. It seems plausible to speculate that when his JPE paper was published, Smith seemingly had no idea that it would mark the start of a line of research that would eventually result in him being appointed as a Nobel laureate. And it seems even less likely that he would have foreseen the extent to which the experimental methods laid out in that 1962 paper would subsequently come to dominate the methodology of researchers working to build adaptive autonomous trading agents by combining tools and techniques from AI, ML, agent-based modelling (ABM), and agent-based computational economics (ACE). Although not a goal stated at the outset, this strand of AI/ML/ABM/ACE research converged toward a common aim: specifying an artificial agent, an autonomous adaptive trading strategy, that could automatically tune its behavior to different market environments, and that could reliably beat all other known automated trading strategies, thereby taking the crown of being the current best trading strategy known in the public domain, i.e., the "dominant strategy". Over the past 20 years the dominant strategy crown has passed from one algorithm to another. Vytelingum's (2006) *AA* strategy, was widely believed to be the dominant strategy, but recent results using contemporary large-scale computational simulation techniques indicate that it does not perform so well as was previously believed from its initial success in small numbers of trials.

Given that humans who are reliably good at trading are generally thought of as being "intelligent" in some reasonable sense of the word, the aim to develop ever more sophisticated artificial trading systems is clearly within the scope of AI research, although some very important early ideas came from the economics literature: a comprehensive review of relevant early research was given in Cliff (1997). Below in Section 2.1 we first briefly introduce eight key publications leading to the development of AA; then describe key aspects of EE market models in Section 2.2; and then discuss each of the eight key publications in more detail in Section 2.3. After that, Section 2.4 summarizes the results of Vach

(2015) and Cliff (2019), which cast doubts on the hitherto apparently resolved issue of which trading agent is best.

2.1. A Brief History of Trading Agents

If our story starts with Smith's 1962 JPE paper, then the next major step came 30 years later, with a surprising result published in the JPE by Gode and Sunder (1993): this popularized a minimally simple automated trading algorithm now commonly referred to as *ZIC*. A few years later two closely related research papers were published independently and at roughly the same time, each written without knowledge of the other: the first was a Hewlett-Packard Labs technical report by Cliff (1997) describing the adaptive AI/ML trading-agent strategy known as the *ZIP* algorithm; the second summarized the PhD thesis work of Gjerstad, in a paper co-authored with his PhD advisor (Gjerstad & Dickhaut 1998), describing an adaptive trading algorithm now widely known simply as *GD*. After graduating his PhD, Gjerstad worked at IBM's TJ Watson Labs where he helped set up an EE laboratory that his IBM colleagues used in a study that generated world-wide media coverage when the results were published by Das *et al.* at the prestigious *International Joint Conference on AI (IJCAI)* in 2001. This paper presented results from studies exploring the behavior of human traders interacting with GD and ZIP robot traders, in a CDA with a Limit Order Book (LOB: explained in more detail in Section 2.2, below), and demonstrated that both GD and ZIP reliably outperformed human traders. Neither GD nor ZIP had been designed to work with the LOB, so the IBM team modified both strategies for their study. A follow-on 2001 paper by Tesouro & Das (two co-authors of the IBM IJCAI paper) described a more extensively *Modified GD* (MGD) strategy, and later Tesouro & Bredin (2002) described the *GD eXtended* (GDX) strategy. Both MGD and GDX were each claimed to be the strongest-known public-domain trading strategies at the times of their publication.

Subsequently, Vytelingum's 2006 thesis introduced the *Adaptive Aggressive* (AA) strategy which, in a major journal paper (Vytelingum *et al.*, 2007), and in later conference papers (De Luca & Cliff 2012a, 2012b), was shown to be dominant over ZIP, GDX, and human traders. Thus far then, AA held the title.

However Vach (2015) presented results from experiments with the *OpEx* market simulator (De Luca, 2015), in which AA, GDX, and ZIP were set to compete against one another, and in which the dominance of AA was questioned: Vach's results indicate that whether AA dominates or not can be dependent on the ratio of AA:GDX:ZIP in the experiment: for some ratios, Vach found AA to dominate; for other ratios, it was GDX. Vach studied only a relatively small sample from the space of possible ratios, but his results prompted Cliff (2019) to exhaustively run through a wide range of differing ratios of four trading strategies (AA, *ZIC*, ZIP, and the minimally simple SHVR strategy described in Section 2.2), doing a brute-force search for situations in which AA is outperformed by the other strategies. The combinatorics of such a search are quite explosive: Cliff

reported on results from over 3.4 million individual simulations of market sessions, and his findings indicated that Vach's observation was correct: AA's dominance does indeed depend on how many other AA traders are in the market; and, in aggregate, AA was routinely outperformed by ZIP and by SHVR. Subsequent research by Snashall (2019) employed the same exhaustive testing method, using a supercomputer to run more than one million market simulations, to exhaustively test AA against IBM's GDX strategy: this again revealed that AA does not always dominate GDX: see Snashall & Cliff (2019) for further discussion.

2.2. On Simulation Models of Markets

Vernon Smith's early experiments were laboratory models of so called *open-outcry trading pits*, a common sight in any real financial exchange before the arrival of electronic trader-terminals in the 1970s. In a trading pit, human traders huddle together and shout out their bids and offers, and also announce their willingness to accept a counterparty's most recent shout. It was a chaotic scene, now largely consigned to the history books. In the closing quarter of the 20th Century, traders moved *en masse* to interacting with each other instead via electronic means: traders "shouted" their offer or bids or acceptances by typing orders on keyboards and then sending those orders to a central server that would display an aggregate summary of all orders currently "shouted" (i.e., quoted) onto the market. That aggregate summary is very often in the form of a *Limit Order Book* or LOB: the LOB summarizes all bids and offers currently live in the market. At its simplest, the LOB is a table of numbers, divided into the *bid side* and the *ask side* (also known as the *offer side*). Both sides of the LOB show the best price at the top, with less good prices arranged below in numeric order of price: for the bid side this means the highest-priced bid at the top with the remaining bid prices displayed in descending order below; and for the ask side the lowest-priced offer is at the top, with the remaining offers arranged in ascending order below. The arithmetic mean of the best bid and best ask prices is known as the *mid-price*, and their difference is the *spread*. For each side of the LOB, at each price on the LOB, the total quantity available is also shown, but with no indication of who the relevant orders came from: in this sense the LOB serves not only to aggregate all currently live orders, but also to anonymize them.

Traders in LOB-based markets can usually cancel existing orders to delete them from the LOB. In a common simple implementation of a LOB, traders can accept the current best bid or best offer by issuing a quote that *crosses the spread*: i.e., by issuing an order that, if added to the LOB, would result in the best bid being at a higher price than the best ask. Rather than be added to the LOB, if a bid order crosses the spread then it is matched with the best offer on the ask side (known as *lifting the ask*), whereas an ask that crosses the spread is matched with the best bid (*hitting the bid*); and in either case a transaction then occurs between the trader that had posted the best price on the relevant side of the LOB, and

the trader that crossed the spread. The price of the resulting transaction is whatever price was hit or lifted from the top of the LOB.

Smith's earliest experiments pre-dated the arrival of electronic trading in real financial markets, and so they can be thought of as laboratory models of open-outcry trading pits: they were simulations of real markets, but were initially not *computer-based* simulations. Even though the much later work by Gode & Sunder, Cliff, Gjerstad & Dickhaut, and Vytelingum all came long after the introduction of electronic LOBs in real markets, these academic studies all stuck with Smith's original methodology, of modelling open-outcry markets (often by essentially operating a LOB with the depth fixed at 1, so the *only* information available to traders is the current best, or most recent, bid and ask prices).

Nevertheless, the studies by IBM researchers (Das *et al.*, 2001; Tesauro & Das, 2001; Tesauro & Bredin 2012), and also the replication and confirmation of AA results by De Luca & Cliff (2011a, 2011b) and by Stotter *et al.* (2013), all used LOB-based market simulators. The IBM simulator *Magenta* seems to have been proprietary to IBM; developed at TJ Watson Labs and not available for third-party use, but De Luca made an open-source release of his *OpEx* simulator (De Luca, 2015) which was subsequently used by Vach (2015) in the studies that prompted our work reported here. Also of relevance here is the *ExPo* simulator described by Stotter *et al.* (2013, 2014): in the work by De Luca, by Vach, and by Stotter *et al.*, Vytelingum's original AA needed modifications to make it work in a LOB-based market environment.

In this paper neither OpEx nor ExPo will be discussed further, but instead we will concentrate on BSE (BSE, 2012; Cliff, 2018) which is another open-source EE market simulator, initially developed as a teaching aid but subsequently used as a platform for research (see, e.g. Le Calvez & Cliff, 2018, Snashall & Cliff 2019). BSE has the advantage of being relatively lightweight (a single Python script of c.2500 lines) and hence readily deployable over large numbers of virtual machines in the cloud. BSE maintains a dynamically updated LOB and also publishes a *tape*, a time-ordered record of all orders that have been executed. It comes with built-in versions of ZIC and ZIP, and also some additionally minimally-simple non-adaptive trading strategies that can be used for benchmarking against other more complex strategies added by the user. One of these, the *Shaver* strategy (referred to in BSE by the abbreviation SHVR) simply reads the best prices on the LOB and, if it is able to do so without risking a loss-making deal then it issues an order that improves the current best bid or best ask by 0.01 units of currency (i.e., one penny/cent), which is BSE's *tick size*, i.e. the minimum change in price that the system allows. Another of the BSE built-in trading strategies is even simpler than SHVR: the *Giveaway* strategy (abbreviated to GVWY) attempts to make no profit at all from trading, and simply posts a bid or offer price that is equal to the limit price assigned to it for that unit. As we will further discuss later in Section 4, when evaluated using conventional market simulation methods like those

used in the papers reviewed below, GVWY can prove to be a profitable strategy: this counterintuitive result is one indication that evaluation via conventional means has some significant limits.

2.3. Eight Key Papers with a Common Methodology

2.3.1. Smith 1962

Although precedents can be pointed to, Smith's 1962 JPE paper is widely regarded as the seminal study in EE. In it he reported on experiments in which groups of human subjects were randomly assigned to be either *buyers* or *sellers*. Buyers were given a supply of artificial money, and sellers were given one or more identical items, of no intrinsic value, to sell. As discussed above, each trader in the market was assigned a private valuation, a secret *limit price*, above which they should not pay when buying and below which they should not accept when selling.

After the allocation of assignments to all subjects, they then interacted via an open-outcry CDA while Smith and his assistants made notes on the sequence of events that unfolded during the experiment: typically, buyers would gradually increase their bid-prices, and sellers would gradually lower their offer-prices (also known as ask-prices) until transactions started to occur. Eventually, usually within a few minutes, the experimental market reached a position in which no more trades could take place, which marked the end of a *trading period* or "trading day" in the experiment; any one experiment typically ran for $n=5-10$ periods, with all the traders being resupplied with money and items-for-sale at the start of each trading period. The sequence of n contiguous trading periods (or an equivalently long single-period experiment with continuous replenishment, as discussed in Section 4.3) is referred to here as one *market session*.

Smith could induce specific supply and demand curves in these experimental markets by appropriate choices of the various limit-prices he assigned to the traders. The market's theoretical *equilibrium price* (denoted hereafter by P_0) is given by the point where the supply curve and the demand curve intersect. Smith found that, in these laboratory CDA markets populated with only remarkably small groups of human traders, transaction prices could reliably and rapidly converge on the theoretical P_0 value despite the fact that each human trader was acting purely out of self-interest and knew only the limit price that he or she had been assigned.

Smith's analysis of his results focused on a statistic that he referred to as α , the root mean square (RMS) deviation of actual transaction prices from the P_0 value over the course of an experiment. In his early experiments, P_0 was fixed for the duration of any one experiment; in later work Smith explored the ability of the market to respond to "price shocks" where, in an experiment of N trading days, on a specific day $S < N$ the allocation of limit prices would be changed, altering P_0 from the value that had been in place over trading periods $1, 2, \dots, S$, to a different value of P_0 that would then remain constant for the rest of the experiment, i.e. in

trading periods $S+1, S+2, \dots, N$. For brevity, in the rest of this paper Smith's initial style of experiments will be referred to as $S'62$ experiments.

2.3.2. ZIC: Gode & Sunder (1993)

Gode & Sunder's 1993 JPE paper used the $S'62$ methodology, albeit with the CDA markets being electronic (a move Smith himself had made in his experiments many years earlier), so each trader was sat at a personal terminal, a computer screen and keyboard, from which they received all information about the market and via which they announced their orders, their bids or offers, to the rest of the traders in the experiment. Gode & Sunder (G&S hereafter) first conducted a set of experiments in which all the traders were human, to establish baseline statistics. Then, all the human traders were replaced with automated trading systems, absolute-zero minimally-simple algo traders which G&S referred to as *Zero Intelligence* (ZI) traders. G&S studied markets populated with two type of ZI trader: ZI-Unconstrained (ZIU), which simply generated random prices for their bids or offers, regardless of whether those prices would lead to profitable transactions or to losses; and ZI-Constrained (ZIC), which also generated random order prices but were constrained by their private limit prices to never announce prices that would lead them to loss-making deals. G&S used fixed supply and demand schedules in each experiment, i.e. there were no price-shocks in their experiments.

Not surprisingly, the market dynamics of ZIU traders were nothing more than noise. But the surprising result in G&S's paper was the revelation that a commonly used metric of market price dynamics known as *allocative efficiency* (AE, hereafter) was essentially indistinguishable between the human markets and the ZIC markets. Because AE had previously been seen as a marker of the degree to which the traders in a market were behaving intelligently, the fact that ZIC traders scored AE values largely the same as humans was a shock. Gode & Sunder proposed that a different metric should instead be used as a marker of the intelligence of traders in the market. This metric was *profit dispersion* (PD, hereafter) which measures the difference between the profit each trader accrued in an experiment, compared to the profit that would be expected for that trader if every transaction in the market had taken place at the market's theoretical equilibrium price P_0 : humans typically showed very low values of PD (which is assumed to be good) while ZIC traders did not. On this basis, G&S argued that PD should be used in preference to AE.

Other researchers were quick to cite G&S's ZIC result, and often used it to support the claim that, given the ZIC traders have no intelligence, then for transaction prices to converge toward the theoretical equilibrium price and/or for a group of traders to score highly on AE, somehow the "intelligence" required to do this must reside within the rules of the CDA market system rather than within the heads of the traders. Strangely, G&S's 1993 paper provides no concrete causal mechanistic explanation of

how their striking ZIC results arise; they describe their methods, and the results observed, but the internal mechanisms that give rise to those results are left as something of a mystery, as if the CDA market was an impenetrable black-box.

A causal mechanistic analysis of markets populated by ZIC traders was subsequently developed in (Cliff 1997), which considered the probability mass functions (PMFs) of prices generated by ZIC buyers and sellers, and the joint PMF of transaction prices in ZIP markets, which is given by the intersection of the bid-price and offer-price PMFs: the shape of the transaction-price PMF is determined by the nature of the supply and demand curves in the market, and (Cliff, 1997) demonstrated that the supply and demand curves in a ZIC market experiment could be arranged so that the expected value of the transaction prices (computable as an integral over the PMF) is identical to the theoretical equilibrium price given by the intersection point of the supply and demand curves. This was why the five ZIC experiments reported in G&S's 1993 paper showed transaction prices that were centered on the theoretical equilibrium price in each case: the supply and demand curves were arranged in such a way that this was the expected outcome. Cliff (1997) showed that with different arrangements of supply and demand curves, such as situations where one or both curves were flat (as had been used in Smith's 1962 JPE paper), the expected price of transactions in ZIP markets could differ considerably from the theoretical equilibrium price, and so transaction prices in those ZIC markets would fail to exhibit human-like convergence toward the theoretical equilibrium value. In these differently-designed experiments, ZIC traders would be revealed for exactly what they are: simple stochastic processes that only coincidentally exhibit human-like market dynamics when the experimenters happen to have chosen to impose the right kind of supply and demand curves. The (Cliff 1997) analysis showed that the level of intelligence in the ZIC traders was insufficient to recreate human-like market dynamics more broadly, and so a more intelligent automated trading strategy was required.

Independently, roughly a decade later, and via a wholly different line of attack Gjerstad & Shachat (2007) also demolished the argument that G&S's ZIC results indicate that the efficiency or intelligence in the market system lies solely within the CDA mechanism. Nevertheless, G&S's 1993 results continue to be widely and uncritically cited by various authors.

2.3.3. ZIP: Cliff (1997)

Taking direct inspiration both from Smith's work and from the ZI paper by G&S, Cliff (1997) developed a ZI trading strategy that used simple machine-learning techniques to continuously adapt the randomly-generated prices quoted by the traders: this strategy, known as ZI-Plus (ZIP) was demonstrated to show human-like market dynamics in experiments with flat supply and/or demand curves: (Cliff 1997) also showed theoretical analyses and empirical results which demonstrated that transaction prices in markets populated only by ZIC traders would

not converge to the theoretical equilibrium price when the supply and/or demand curves are flat (or, in the language of microeconomics, “perfectly elastic”). EE studies in which the supply and/or demand curve was flat had previously been reported by Smith and others, but G&S had not explored the response of their ZIC traders to this style of market. The work in (Cliff, 1997) involved no human traders: all the focus was on markets populated entirely by autonomous agents, by ZIP traders. In total there are results from fewer than 1,000 simulated market sessions reported on in (Cliff 1997). In all other regards (Cliff 1997) continued the S’62 tradition: key metrics were Smith’s α , AE, and PD, and the focus on *homogenous* markets continued the tradition established by Smith (who studied all-human markets) and by G&S (who studied markets homogeneously populated with either human, ZIU, or ZIC traders).

2.3.4. GD: Gjerstad & Dickhaut (1997)

Gjerstad’s PhD studies of price formation in CDA markets also involved creating an algorithm that could trade profitably by adapting its behavior over time, in response to market events (Gjerstad & Dickhaut, 1998). In contrast to the ZI work, Gjerstad’s trading algorithm uses frequentist statistics, gradually constructing and refining a *belief function* that estimates the likelihood for a bid or offer to be accepted in the market at any particular time, mapping from price of the order to its probability of success. Gjerstad did not explicitly name his strategy, but it has since become widely known as the GD strategy. In all other regards, as with Cliff (1997) and G&S (1993), Gjerstad’s work was firmly in the S’62 tradition: homogenous markets of GD traders interacting in a CDA, buying and selling single items, with the metrics being Smith’s α , AE, and PD. In a later paper, Gjerstad (2003) made some refinements to the GD algorithm and named it *HBL* (Heuristic Belief Learning), although the original GD remains by far the most cited.

2.3.5. MGD: Das *et al.* (2001)

In their landmark 2001 IJCAI paper, IBM researchers Das, Hanson, Kephart, & Tesauro studied the performance of GD and ZIP in a series of EE market experiments where, for the first time ever in the same market, some of the traders were robots while others were human (recall that the earlier work of Smith, of G&S, of Cliff, and of Gjerstad & Dickhaut had all studied homogeneous markets: either all-human or all-robot). Das *et al.* used a LOB-based market simulator called *Magenta*, developed by Gjerstad, and ran a total of six experiments, six market sessions, in which humans and robots interacted and where there were three shock-changes to P_0 , i.e. four phases in any one experiment, each phase with a different P_0 value that was held static over that phase. The surprising result in this paper was that robot trading strategies could consistently outperform human traders, by significant margins: a result that attracted worldwide media attention. Both GD and ZIP outperformed human traders, and in the six experiments reported by Das *et al.* the results from the

two robot strategies are so similar as to not obviously be statistically significant. A subsequent paper by IBM’s Tesauro & Das (2001), reported on additional studies in which a *Modified GD* (MGD) strategy was exhibited what the authors described in the abstract of their paper as “...*the strongest known performance of any published bidding strategy*”.

2.3.6. GD: Tesauro & Bredin (2002)

Extensions to MGD were reported by IBM researchers Tesauro & Bredin (2002) at AAMAS 2002. This paper described extensions to MGD, using dynamic programming methods: the extended version was named *GDX* and its performance was evaluated when competing in heterogeneous markets with ZIP and other strategies. Tesauro and Bredin reported that GDX outperformed the other strategies and claimed in the abstract of their paper that GDX “...*may offer the best performance of any published CDA bidding strategy*.”

2.3.7. AA: Vytelingum (2006)

Vytelingum developed AA and documented it in full in his PhD thesis (2006) and in a major paper (Vytelingum *et al.*, 2008). The internal mechanisms of AA are described in greater detail in Section 3 of this paper. Although Vytelingum’s work came a few years after the IBM publications, the discussion within Vytelingum’s publications is phrased very much in terms of the S’62 methodology: the P_0 value in his AA experiments was either fixed for the duration of each market session, or was subjected to a single “price shock” partway through the session (as described in Section 2.3.1); and again the primary metrics studied are Smith’s α , AE, and PD. Vytelingum presented results from heterogeneous market experiments where AA, GDX, and ZIP traders were in competition, and the published results indicated that AA outperformed both GDX and ZIP by small margins. In total, results from c.25,000 market sessions are presented in (Vytelingum *et al.*, 2008).

2.3.8. AA Dominates: De Luca & Cliff (2011)

As part of the research leading to his 2015 PhD thesis, De Luca used his LOB-based *OpEx* market simulator system (De Luca, 2012) to study the performance of AA in heterogeneous market experiments where some of the traders were AA, some were other robot strategies such as ZIP, and some were human traders sat at terminals interacting with the other traders (human and robot) in the market via the OpEx GUI, in the style introduced by the IBM team in their IJCAI 2001 paper. De Luca & Cliff (2011a) had previously published results from comparing GDX and AA in OpEx, at ICAART-2011; and the first results from AA in human-agent studies were then published in a 2011 IJCAI paper (De Luca & Cliff, 2011b), in which AA was demonstrated to dominate not only humans but also GDX and ZIP. For consistency with what was by then a well-established methodology, in De Luca’s experiments the P_0 value was static for sustained periods with occasional “shock” step-changes to different values. Continuing the tradition

established by the IBM authors, the abstract of (De Luca & Cliff 2011b) claimed supremacy for AA: “We... demonstrate that AA’s performance against human traders is superior to that of ZIP, GD, and GDX. We therefore claim that... AA may offer the best performance of any published bidding strategy”. And, until the publication of Vach (2015), that claim appeared to be plausibly true.

2.4. Actually, AA doesn't dominate

Vach's 2015 Master's Thesis tells the story of his design of a new trading strategy based on ZIP and called ZIPOJA, which he then tested against AA, GDX, and ZIP. The testing revealed that ZIPOJA did not consistently outperform any of the three pre-existing strategies. But, in the course of that testing, as Vach checked and calibrated his implementations of the three pre-existing strategies, he found that AA could fail to dominate ZIP or GDX, depending on the proportions of the two strategies in the market: this runs counter to the established story that AA is the best-performing strategy. Tables 6.2 and 6.3 on p.47 of Vach's thesis show results from tests in which the performance of two trading strategies were tested in trials with proportions of the two trader strategies set at 6:0, 5:1, 4:2, 3:3, 2:4, 5:1, and 0:6. The ratios 6:0 and 0:6 are homogeneously populated by one strategy or the other and hence are of little interest, because that one strategy necessarily dominates in those markets. In Vach's Table 6.2, AA is outperformed by ZIP when the ZIP:AA ratio is 1:5 – i.e, if one in six of the traders in the market are ZIP with the rest AA, then the ZIP traders will outperform the AAs: the efficiency of the ZIP traders was 99.5% while the efficiency of the AAs was 88.5%. In Vach's Table 6.3, AA is outperformed by GDX when the GDX:AA ratio is 3:3, 2:4, and 1:5.

Vach then performed three-way simulations systematically varying the ratios of AA:GDX:ZIP over all possible permutations and, in his Fig.6.1i (p.53) he shows a 2D simplex diagram which summarizes those results: a 28-node regular isometric mesh is drawn over the surface of the simplex as a co-ordinate frame, and AA is the dominant strategy in only 11 of those 28 nodes. Each of the three strategies is by definition dominant at the node representing a homogeneous ratio (i.e., either 1:0:0 or 0:1:0 or 0:0:1), so AA actually only dominates at 10 of the 25 nodes where it is actually contesting with the other two strategies: ZIP dominates one of the remaining nodes, and GDX dominates the remaining 14.

In a final four-way study, with AA, GDX, ZIP, and ZIPOJA competing against each other, Vach (2015, Table 6.7, p.60) declares GDX the overall winner although in that experiment the scores of GDX and AA are sufficiently close that the difference between the two is not obviously significant. Nevertheless, it is undeniable that in Vach's four-way study AA fails to clearly dominate. To the best of our knowledge, Vach's results are the first exhaustive study of AA's performance as the number and proportion of competitor strategies is systematically varied, and he was the first to demonstrate that AA is in fact not the best-performing strategy.

Cliff (2019) set out to replicate and extend Vach's results, using a finer-grained analysis, varying the proportions of AA, SHVR, ZIP, and ZIC, and also studying the effects of altering other aspects of the experiment design such as whether the replenishment of assignments to the traders is periodic or continuous-stochastic (see e.g. Cliff & Preist 2001); and whether the equilibrium price P_0 is largely constant with occasional shock-jumps, or continuously varying according to price-movements taken from real-world markets. Cliff's results from conventional S'62-style experiments, with periodic replenishment and largely constant, confirmed the established view: when AA was tested in the kind of simple market environment as has traditionally been used in the previous literature, AA scored just as well as well-known other trading strategies and was not dominated by them. But merely by altering the nature of the market environment to have continuous stochastic replenishment (which is surely what happens in real markets) and to have the equilibrium price P_0 continuously varying over time (which is also surely what happens in real markets), Cliff's results from AA were very poor indeed. Cliff (2019) concluded that AA's success as reported in previous papers seemed to be largely due to the extent to which AA's internal mechanisms are designed to fit exactly the kind of experiment settings first introduced by Vernon Smith: AA is very well suited to situations in which all assignments are issued to all traders simultaneously, and in which P_0 remains constant for sustained periods of time, with only occasional step-change “shocks”. Real markets are not like this, and when AA is deployed in the more realistic market setting provided by BSE, Cliff (2019) demonstrated that AA's dominance disappears.

Cliff (2019) did not present results of exhaustive testing of AA against GDX, but Snashall's (2019) subsequent thesis did so, running more than one million simulation experiments. Snashall demonstrated that actually, even in the S'62 style of experiment that AA was first tested in, if AA is evaluated exhaustively in BSE across a wide range of proportions, then AA can be outperformed by GDX.

However, as Snashall (2019) argues, GDX's apparent superiority in some situations may itself be illusory, because of the computationally intensive nature of the GDX algorithm. In Snashall's study, GDX (which extends the original GD algorithm with techniques from dynamic programming) took roughly ten times longer than AA or ZIP to compute its response to a change in the LOB, every time the LOB data altered. Such a disproportionately long time spent “thinking” would most likely be a serious impediment to deploying GDX in today's electronic markets where speed of reaction time is a critical factor in determining the success or failure of an automated trading system: see Snashall & Cliff (2019) for further discussion.

3. MARKET MICROSTRUCTURAL ISSUES

The story told in Sections 2.1 to 2.4 should be of concern to anyone who cares about the use of simulation-based

evaluation in empirical science: a sequence of papers on performance of automated traders in simulated markets is published in leading international peer-reviewed AI conferences and journals, each building on and extending the work that had gone before it, and in which a set of apparently reasonable conclusions are drawn that, roughly a decade later, more sophisticated simulation studies call into serious question.

It is very important to note here that I am not calling into question the honesty or professionalism of the researchers involved in this sequence of papers. Each of the papers reviewed above was, as far as I know, prepared in good faith and then subjected to high standards of peer-review. There is no reason to doubt any of the results published in any of those papers. The issue is not with the results or the researchers, but rather with the wider methodological context, the "spirit of the time", within which these studies were conducted.

So, for example, there is no reason to doubt that Vytelingum's (2006, Vytelingum *et al.* 2008) published results showing AA outperforming GDX are genuine. They can be trusted as a fair representation of what happens when AA is placed in a S'62-style market simulator (as had also been used in Cliff's 1997 ZIP publication) in which all assignments are reissued periodically and there is no LOB. But as the results of Vach (2015), Cliff (2019), Snashall (2019), and Snashall & Cliff (2019) show, as soon as AA is deployed in a more realistic market simulator (such as BSE, in which there is a fully working LOB, and in which assignment updates arrive in a continuous random stream rather than in neat periodic bursts to all traders simultaneously), AA's dominance disappears.

AA, just like ZIP, GD, MGD, and GDX, has a simplicity in its specification that is a reflection of the simplicity of the underlying market simulators in which it was developed and tested. For example, if all traders' individual assignments are issued simultaneously in periodic bursts at the start of each trading period (each "day", as in the S'62 experiments), and each trader is given only one unit to buy or sell per assignment, then it makes perfect sense to also clear the LOB at the start of each trading period (also as in S'62) and hence the traders in the market never have to deal with order *cancellations* (i.e., where an existing limit order visible on the LOB is cancelled by the trader who issued that order, and the LOB is updated to reflect that change). As soon as the issuing of assignments is switched to be a continuous stochastic stream, occasions will arise in which a trader receives a new assignment and hence has to cancel a limit order that was previously issued to the exchange – e.g. because the new assignment has a radically different limit price to the previous one. As soon as cancellations are a routine occurrence, all of the trading algorithms described here need to be amended or extended to correctly distinguish between changes in the LOB that are the result of actual transactions taking place, and changes that are the result of cancellations.

So the real issue here instead seems (now, with hindsight) to be the extent to which the simplistic

underlying market simulators were trusted by the entire community of researchers: the people doing the actual work and writing the papers; the people doing the reviewing; and the people who subsequently cited the published papers. (And, for the avoidance of doubt, I myself fall into all three of those categories: so, *mea culpa*). Smith's 1962 experiments used a non-computer-based manual simulation of the CDA found in real financial markets; once the S'62 methodology was established, it was natural for subsequent papers to use close copies of that methodology, to ease the comparison of new results with those already published. Hence it is natural that first G&S (1993), then Cliff (1997) and Gjerstad & Dickhaut (1998), and latterly Vytelingum (2006, 2008) all used essentially the same methodology.

Similarly, as Snashall (2019) has highlighted, the commonly-used simple single-threaded simulation approximations to what in real life is a parallel and asynchronous distributed system mean that the much longer compute-times required by GDX have not previously been highlighted by other researchers. Many real-world trading strategies have to be sensitive not only to the price that is agreed for a transaction, but also how much time is taken in arriving at an agreed price. While Kaplan's *Sniper* strategy (Rust *et al.* 1992) and Gjerstad's (2003) *HBL* strategy are both time-sensitive to varying degrees, these two strategies are the exception rather than the rule: yet the strategies described in the key papers reviewed here all essentially ignore timing issues and concentrate only on price. A very recent paper by Miles & Cliff (2019) discusses issues of latency and time-sensitivity in more detail.

And, in addition to considerations of price and time, there is a third factor that any real-world trading agent is likely to pay attention to: the quantity available at any particular price (also referred to as the *size* or the *volume*). In particular, if there is a much larger quantity available at the best price on one side of the LOB in comparison to the other side, that imbalance can be a strong indication that the market price is likely to move in the near future: excess supply pushes prices down; excess supply pushes price up. Market shocks can occur not only via sudden changes in prices (with quantities available remaining the same), but also by sudden changes in the quantities available (with the prices staying unchanged, initially at least). As with time-sensitivity, size-sensitivity was essentially ignored by the authors of the key papers reviewed above. A very recent publication by Church & Cliff (2019) discusses size issues in more detail.

All of these concerns are essentially *microstructural*: the details of how the assignments are distributed to the traders; how the traders' orders are processed by the exchange and what data the exchange then publishes to the traders for them to react to and act upon; and which factors of the orders on the LOB matter to traders (i.e.: price, size, and timing). However, there are also broader issues, characterized here as *macrostructural*, discussed in the next section.

4. MARKET MACROSTRUCTURAL ISSUES

Smith's 1962 experiments are a fair approximation to an open-outcry *trading pit*, which was a common sight in financial exchanges prior to their wholesale automation in the last thirty-five years: a group of traders gathered in close physical proximity, shouting and gesturing at each other while trying to find a counterparty to trade with. It seems entirely reasonable for Smith to have set up a (manual) simulation of the financial exchanges of his day for his laboratory studies of the CDA, but that was more than half a century ago; surely today's market simulations should be structured in such a way that they are closer to the realities of today's actual financial markets?

The market simulators used in all the studies reviewed in Section 2 each essentially followed the S'62 pattern: some population of individual traders compete within a given market mechanism, and summary statistics (such as α , AE, and PD) are computed across the population of traders. So, in that sense, they are also simulations of a single trading pit. But the reliance on computing population-level statistics gives rise to a result that makes little sense: the minimally simple GVWY trading strategy, introduced in Section 2.2, seeks no profit at all and yet when its performance is measured in a population of traders, using standard population-level statistics, it can routinely outperform other more intelligent traders. In particular, GVWY traders (which always and immediately post orders on the LOB with prices equal to their private limit price, thereby guarantee them zero profit if their order is accepted by a counterparty) can often score an average profit per trader that is as good as, or better than, the average profit per trader of supposedly intelligent adaptive trading strategies such as AA or ZIP.

This counterintuitive result does have a rational explanation: an individual GVWY buyer(seller) can do well against any other type of trader if its private limit price L_G is sufficiently above(below) the best price on the ask(bid) side of the LOB. For example, say that you have a GVWY buyer G with limit price $L_G = 150$, and the LOB's current best bid is 90 and best ask is 100, with that best ask coming from a seller whose private limit price is $L_S = 80$: the GVWY trader G is polled to provide a quote and bids 150. This crosses the spread (because $150 > 100$), and so the transaction goes through at price of current best ask, i.e. 100. That gives the seller who posted the ask a profit of $100 - L_S = 20$, and the GVWY gets a profit of $L_G - 100 = 50$. So, even though the GVWY order submitted to the market would have generated no profit for that trader *if it had been accepted at the indicated price*, instead the order crosses the spread and so the GVWY trader makes a nonzero profit equal to the difference between the best price on the counterparty side of the LOB. As an individual GVWY can never make a negative profit, its average score will be computed from the sum of a sequence of profits that are either zero or positive, and so the average profit per GVWY trader will often be a positive value. This shows how GVWY can make profit. Exactly how much profit GVWY can make depends quite a lot on the Supply/Demand schedule in the market at that time, and on the mix of other strategies that it finds

itself competing with. But, crucially, adaptive strategies such as AA, ZIP, or GDX, may take some time to adjust their prices to the point where they successfully identify a counterparty to trade with, and in the time it takes an adaptive strategy to find a counterparty to trade with, a GVWY trader might have executed a sequence of several transactions. So although the GVWY average profit per *trade* may be small, and although its transactions may go through at prices some way distant from P_0 (so its score for a may be poor in comparison to other strategies) by the end of an experiment its average profit per *trader* may actually be better than those of the adaptive "intelligent" trading strategies. Whether this happens or not depends on the proportions of the different trading strategies and the nature of the experimental market's supply and demand and updates to traders' assignments, but the fact that it can happen at all gives some pause for thought: in principle, when evaluated using the conventional techniques, GVWY could out-perform any of the more sophisticated trading strategies.

This calls into question the measuring of the performance of trading strategies at the population level: when monitored across a population, GVWY can come out as a good/dominant strategy because it's constrained to never post a quote price the wrong side of its limit price and so it never enters into loss-making deals. GVWY never loses, but occasionally wins big: aggregate this over lots of individuals doing lots of deals and GVWY's apparent profits can start to add up. Then couple that with the fact that GVWY is wholly incautious: it "goes for the deal" as fast as it can and either gets some profit or nothing, but then gets another assignment to deal with. Adaptive algorithms like ZIP and AA and MGD/GDX all spend a bit of time doing their adapting – they can often commence by quoting a price some distance from equilibrium and then gradually edge towards it, which means their count of trades per unit time is much lower than GVWY, which counts against them when they're ranked on profit-per-unit-time. Someone who studies trading strategies only in the conventional S'62 style of experiment might reasonably conclude that GVWY is a good strategy.

But this is not likely to go down well in a real trading situation, because real traders do not evaluate their performance at the population level: they are typically ruthlessly self-interested, and care only about the profit that they make as an individual. This is a key macro-level issue: while academic economists seem primarily interested in population-level statistics, individual traders tend instead to be singularly focused on their own personal profit-and-loss (P&L), and not much else.

From this it seems plausible to conjecture that the progression of ZIC-ZIP/GD-MGD/GDX-AA described in Section 2 may have been a result of people testing their trading agents in unrealistic environments and measuring them with the wrong metrics. If the underlying simulations had been more realistic, the tale of which strategy comes out as best (or whether *any* one strategy ever comes out as consistently better than the others) could plausibly have been a different story. And the best

way of testing that conjecture is to start building and using market simulators that more accurately reflect modern financial markets.

For instance, in many contemporary financial markets, the key players are no longer individual traders, single humans buying or selling, but instead the market is populated by some number N of trading *entities* which (although they could in principle still be individual humans) are more likely to be trading institutions such as banks or fund management companies. In most such trading entities, there will be a currently trusted, established trading system known as the *production system* (because it is deployed in live trading, in production) which is made up of *one or more* distinct trading strategies working in concert. Any one instance of a production system is likely to be well-suited to the market circumstances that prevailed at the time that it was introduced, but market circumstances rarely stay the same for long and so what is a profitable production system this week or this month may be generating much less income next week or next month, and for that reason it is common for trading entities to always also be working on a *development system*, again a set of *one or more* coordinated trading strategies, which is intended to be deployed as the eventual replacement for the current production system. Practitioners working with such coupled pairs of trading systems often use the abbreviations "prod" for the production system and "dev" for the development system: we'll use them in the rest of this paper too.

In the previous paragraph the phrase "one or more" was emphasized when introducing the notion of prod and dev systems to highlight the issue that many real-world trading entities now routinely employ what are commonly known as *algo-wheels*: systems that, for each order to be executed, automatically select one particular algorithmic trading strategy (the *algo*) from among a set of potentially applicable strategies (the *wheel*), depending on the nature of the order and the market conditions prevailing at the time. One of the first algo-wheel offerings to achieve widespread notice was the *Algo Switching Engine* from Pipeline Trading Systems LLC: see e.g. Stephens & Waelbroeck (2009).

A more accurate simulation model of contemporary automated markets would capture all of the above macrostructural factors: i.e., have the simulator allow for N trading entities, each trying to make profit from its current set of one or more prod trading strategies while also each working on one or more dev trading strategies that are intended to improve upon the current prod strategies. As in real markets, each entity should be able to monitor only its own (P&L) on its various strategies; for any one entity the technical details of its strategies and their individual P&L streams are private, not disclosed to any other trading entities, and hence any one entity has no knowledge of the technical details or the profitability of the trading strategies being deployed by the various other entities that it is competing with in the market. A simulator constructed in this way would be a major step towards providing a test-bed for simulation

studies in which new trading strategies, expressed as algorithms, can be evaluated in environments that are reasonable approximations of today's financial markets. In the next Section, I describe how the freely-available open-source BSE market simulator has been extended to meet these needs.

5. BSE2: SIMULATING MODERN MARKETS

The BSE financial-market simulator (BSE 2012, Cliff 2018) has undergone major refactoring and extension, resulting in a "Version 2" of BSE, known as BSE2. The source code for this, written in Python, is being made freely available via the BSE GitHub repository.

BSE2 models current real-world financial markets in which some number N of profit-seeking legal entities each deploy proprietary automated trading systems. Each individual entity maintains an internal population of trading strategies, which it can select among for each order that is executed. In the simplest case, BSE2 can be configured so that each entity has an internal strategy population of size 1, so each entity is running a single specific strategy: this implements the S'62 style of modelling witnessed in the papers reviewed in Section 2.

To simulate a modern market scenario, each of the N entities simultaneously runs at least two trading strategies: one or more operational *prod* strategies which are to some extent trusted; and one or more *dev* strategies which are to some extent experimental improvements on that entity's current production strategies. The N populations of strategies, one population per entity, manifestly invite comparison with research studying meta-population dynamics. Taking the minimal case of each entity running only one prod and one dev strategy, as a first approximation the strategy innovation and improvement process within any one entity can be usefully modelled as a (1+1) Evolution Strategy (ES) optimization system: see e.g. Beyer & Schwefel 2002). The overall system is in fact *co-evolutionary* because the profitability (or, in the language of ESs, the *fitness function*) for any one entity's trading activity is defined at any one time largely by the nature of the $N-1$ other entities' current trading strategies, which are simultaneously in the process of adapting. Preliminary simulation results studying co-adaptive dynamics in metapopulations of traders have been reported by Witchett (2004) using a version of the original ZIP market simulator (Cliff, 1997); and more recently by Hukerikar (2019) who worked with the original (Version-1) BSE studying the population dynamics of various simple trading strategies based on parameterized versions of ZIC, SHVR and GVWY, three of the built-in strategies in BSE.

BSE2 enables studying the evolutionary optimization of sets of parameter values for pre-established algorithms (see e.g. Cliff, 2009), operating in modern market settings; and could in further work be additionally extended to allow arbitrary trading strategies to be evolved and adapted via a genetic programming approach, revisiting early work such as that by Andrews

& Prager (1994), but in the modern market contexts provided by BSE2.

The various BSE2 source-code files can trivially be combined into a single Python script which can then readily be copied and launched across multiple virtual machines (VMs) from a commercial cloud service provider, so that many experiments can be conducted in parallel across a bank of VMs: the variability of market systems and the nature of the exhaustive testing required to establish rigorous results is likely forever more to require many millions of individual market sessions to be simulated, but this is a task that is *embarrassingly parallel* (i.e., when N i.i.d. simulation sessions are split across M machines, so that each machine is responsible for N/M simulated sessions, the speed-up factor is directly proportional to M).

The BSE2 GitHub repository, like the original BSE repository first created in 2012, offers not only the source-code for the BSE exchange simulator itself, but also the source-code for the trading algorithms ZIC, ZIP, GDX, and AA that were introduced in the review of key literature in Section 2, along with the source-code for simpler test strategies such as SHVR and GVWY. This served an important function of providing reference implementations for frequently-cited trading strategies, and is only notable because the authors of the papers introducing what were proposed (at the time) as world-leading algorithms such as GDX and AA did not provide any sample code: the published descriptions of those two algorithms are both written only as narrative English text with occasional mathematical equations, and hence are open to some variation in interpretation. If the scientific study of trading strategies is to proceed smoothly, there is a need not only for a common and open freely-available up-to-date simulator for financial markets (as is now provided by BSE2) but also for freely-available open-source reference implementations of the major algorithms in the published literature (a point that was made forcefully over a decade ago by Toft, 2007), and of any other algorithms published in future.

6. SUMMARY

The BSE financial-market simulator (BSE 2012, Cliff 2018) has undergone major refactoring and extension, and is now known as BSE2. The BSE2 source-code is freely available on GitHub. BSE2 offers the facility not only for continuing to conduct experiments in the style of Smith (1962) but also allows trading strategies to be tested in simulations of modern-day markets where multiple trading entities, each in principle running more than one trading strategy at any one time, co-adapt via their interactions in the market. Because of the increased realism of BSE2, understanding the co-adaptive dynamics of market scenarios simulated in BSE2 is likely to help further our understanding of the dynamics of real-world financial markets, which are themselves inherently co-adaptive. Doing such work rigorously requires a shift in mind-set from the old view (prevalent in the key papers reviewed in Section 2) that a few tens of thousands of simulation sessions is sufficient to

establish trusted results, to a new revised norm where it is routine for results to be published from tens or hundreds of millions of such experiments: the ready availability of cheap cloud computing services makes such an increase in CPU-cycles expended both practicable (because of the embarrassingly parallel nature of the simulations) and affordable (because of the economies of scale that have driven the development of commercial cloud service provisioning). As with its predecessor version, the BSE2 simulator has been made freely available via a standard MIT Open Source Licence on the GitHub public repository, and this includes not only the source-code for the exchange but also the source-code for various well-known trading strategies. The intention is that the BSE2 codebase becomes a common platform that is collectively refined and extended by the community of researchers interested in testing trading strategies in agent-based models of current real-world financial networks. If that does happen, then the effort expended in getting it this far will have been worthwhile.

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